Essential Question How can proportions help you decide when things are "fair?"

The Meaning of a Word ● Proportional

When you work toward a goal, your success is usually **proportional** to the amount of work you put in.

An equation stating that two ratios are equal is a **proportion**.

1 ACTIVITY: Determining Proportions

Work with a partner. Tell whether the two ratios are equivalent. If they are not equivalent, change the second day to make the ratios equivalent. Explain your reasoning.

a. On the first day, you pay $5 for 2 boxes of popcorn. The next day, you pay $7.50 for 3 boxes.

First Day
$$\frac{\$5.00}{\$7.50} \overset{?}{=} \frac{2 \text{ boxes}}{3 \text{ boxes}}$$
Next Day

b. On the first day, it takes you 3 hours to drive 135 miles. The next day, it takes you 5 hours to drive 200 miles.

First Day
$$\frac{3 \text{ h}}{5 \text{ h}} \overset{?}{=} \frac{135 \text{ mi}}{200 \text{ mi}}$$
Next Day

c. On the first day, you walk 4 miles and burn 300 calories. The next day, you walk 3 miles and burn 225 calories.

First Day
$$\frac{4 \text{ mi}}{3 \text{ mi}} \overset{?}{=} \frac{300 \text{ cal}}{225 \text{ cal}}$$
Next Day

d. On the first day, you download 5 songs and pay $2.25. The next day, you download 4 songs and pay $2.00.

First Day
$$\frac{5 \text{ songs}}{4 \text{ songs}} \overset{?}{=} \frac{\$2.25}{\$2.00}$$
Next Day

2 ACTIVITY: Checking a Proportion

Work with a partner.

a. It is said that "one year in a dog's life is equivalent to seven years in a human's life." Explain why Newton thinks he has a score of 105 points. Did he solve the proportion correctly?

$$\frac{1 \text{ year}}{7 \text{ years}} \overset{?}{=} \frac{15 \text{ points}}{105 \text{ points}}$$

b. If Newton thinks his score is 98 points, how many points does he actually have? Explain your reasoning.
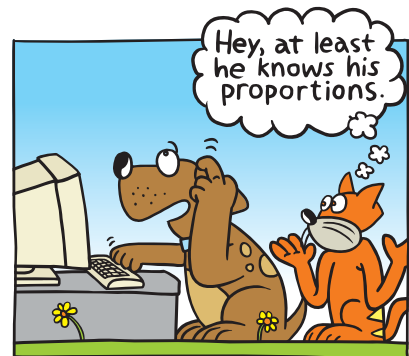
"I got 15 on my online test. That's 105 in dog points! Isn't that an A+?"

3 ACTIVITY: Determining Fairness

Work with a partner. Write a ratio for each sentence. If they are equal, then the answer is "It is fair." If they are not equal, then the answer is "It is not fair." Explain your reasoning.

a.
| You pay $184 for 2 tickets to a concert. | & | I pay $266 for 3 tickets to the same concert. | ▶ Is this fair? |

b.
| You get 75 points for answering 15 questions correctly. | & | I get 70 points for answering 14 questions correctly. | ▶ Is this fair? |

c.
| You trade 24 football cards for 15 baseball cards. | & | I trade 20 football cards for 32 baseball cards. | ▶ Is this fair? |

What Is Your Answer?

4. Find a recipe for something you like to eat. Then show how two of the ingredient amounts are proportional when you double or triple the recipe.

5. **IN YOUR OWN WORDS** How can proportions help you decide when things are "fair?" Give an example.

Practice ▶ Use what you discovered about proportions to complete Exercises 17–22 on page 114.

Key Idea

Proportions

Words A **proportion** is an equation stating that two ratios are equivalent. Two quantities that form a proportion are **proportional**.

Numbers $\dfrac{2}{3} = \dfrac{4}{6}$ The proportion is read "2 is to 3 as 4 is to 6."

EXAMPLE 1 Determining Whether Ratios Form a Proportion

Tell whether the ratios form a proportion.

a. $\dfrac{4}{10}$ and $\dfrac{10}{25}$

Compare the ratios in simplest form.

$$\dfrac{4}{10} = \dfrac{4 \div 2}{10 \div 2} = \dfrac{2}{5}$$

$$\dfrac{10}{25} = \dfrac{10 \div 5}{25 \div 5} = \dfrac{2}{5}$$

The ratios are equivalent.

So, $\dfrac{4}{10}$ and $\dfrac{10}{25}$ form a proportion.

b. $\dfrac{6}{4}$ and $\dfrac{8}{12}$

Compare the ratios in simplest form.

$$\dfrac{6}{4} = \dfrac{6 \div 2}{4 \div 2} = \dfrac{3}{2}$$

$$\dfrac{8}{12} = \dfrac{8 \div 4}{12 \div 4} = \dfrac{2}{3}$$

The ratios are not equivalent.

So, $\dfrac{6}{4}$ and $\dfrac{8}{12}$ do not form a proportion.

On Your Own

Tell whether the ratios form a proportion.

Now You're Ready
Exercises 5–16

1. $\dfrac{1}{2}, \dfrac{5}{10}$ **2.** $\dfrac{4}{6}, \dfrac{18}{24}$ **3.** $\dfrac{10}{3}, \dfrac{5}{6}$ **4.** $\dfrac{25}{20}, \dfrac{15}{12}$

Multi-Language Glossary at BigIdeasMath com.

Key Ideas

Cross Products

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called **cross products**.

Cross Products Property

Words The cross products of a proportion are equal.

Numbers
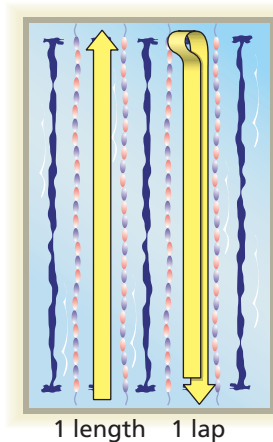
$$\frac{2}{3} = \frac{4}{6}$$

$2 \cdot 6 = 3 \cdot 4$

Algebra

$$\frac{a}{b} = \frac{c}{d}$$

$ad = bc$,
where $b \neq 0$ and $d \neq 0$

EXAMPLE 2 **Identifying Proportional Relationships**

You swim your first 4 laps in 2.4 minutes. You complete 16 laps in 12 minutes. Is the number of laps proportional to your time?

Method 1: Compare unit rates.

$\div 4$

$$\frac{2.4 \text{ min}}{4 \text{ laps}} = \frac{0.6 \text{ min}}{1 \text{ lap}}$$

$\div 4$

$\div 16$

$$\frac{12 \text{ min}}{16 \text{ laps}} = \frac{0.75 \text{ min}}{1 \text{ lap}}$$

$\div 16$

> The unit rates are not equivalent.

1 length   1 lap

So, the number of laps is not proportional to the time.

Method 2: Use the Cross Products Property.

$$\frac{2.4 \text{ min}}{4 \text{ laps}} \overset{?}{=} \frac{12 \text{ min}}{16 \text{ laps}} \qquad \text{Test to see if the rates are equivalent.}$$

$$2.4 \cdot 16 \overset{?}{=} 4 \cdot 12 \qquad \text{Find the cross products.}$$

$$38.4 \neq 48 \qquad \text{The cross products are not equal.}$$

So, the number of laps is not proportional to the time.

On Your Own

5. You read the first 20 pages of a book in 25 minutes. You read 36 pages in 45 minutes. Is the number of pages read proportional to your time?
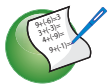
✓ Vocabulary and Concept Check

1. **VOCABULARY** What does it mean for two ratios to form a proportion?

2. **VOCABULARY** What are two ways you can tell that two ratios form a proportion?

3. **OPEN-ENDED** Write two ratios that are equivalent to $\frac{3}{5}$.

4. **WHICH ONE DOESN'T BELONG?** Which ratio does *not* belong with the other three? Explain your reasoning.

$$\frac{4}{10} \qquad \frac{2}{5} \qquad \frac{3}{5} \qquad \frac{6}{15}$$

Practice and Problem Solving

Tell whether the ratios form a proportion.

① 5. $\frac{1}{3}, \frac{7}{21}$

6. $\frac{1}{5}, \frac{6}{30}$

7. $\frac{3}{4}, \frac{24}{18}$

8. $\frac{2}{5}, \frac{40}{16}$

9. $\frac{48}{9}, \frac{16}{3}$

10. $\frac{18}{27}, \frac{33}{44}$

11. $\frac{7}{2}, \frac{16}{6}$

12. $\frac{12}{10}, \frac{14}{12}$

13. $\frac{27}{15}, \frac{18}{10}$

14. $\frac{4}{15}, \frac{15}{42}$

15. $\frac{76}{36}, \frac{19}{9}$

16. $\frac{49}{77}, \frac{38}{57}$

Tell whether the two rates form a proportion.

② 17. 7 inches in 9 hours; 42 inches in 54 hours

18. 12 players from 21 teams; 15 players from 24 teams

19. 440 calories in 4 servings; 300 calories in 3 servings

20. 120 units made in 5 days; 88 units made in 4 days

21. 66 wins in 82 games; 99 wins in 123 games

22. 68 hits in 172 at bats; 43 hits in 123 at bats

23. **FITNESS** You can do 90 sit-ups in 2 minutes. Your friend can do 135 sit-ups in 3 minutes. Are these rates proportional? Explain.

24. **HEARTBEAT** Find the heartbeat rates of you and your friend. Do these rates form a proportion? Explain.

	Heartbeats	Seconds
You	22	20
Friend	18	15

Tell whether the ratios form a proportion.

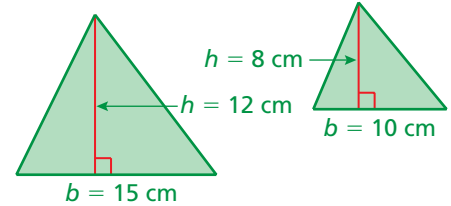**25.** $\dfrac{3}{8}, \dfrac{31.5}{84}$

26. $\dfrac{14}{30}, \dfrac{75.6}{180}$

27. $\dfrac{2.5}{4}, \dfrac{7}{11.2}$

28. **PAY RATE** You earn $56 walking your neighbor's dog for 8 hours. Your friend earns $36 painting your neighbor's fence for 4 hours.

    a. What is your pay rate?

    b. What is your friend's pay rate?

    c. Are the pay rates equivalent? Explain.

29. **GEOMETRY** Are the ratios of h to b in the two triangles proportional? Explain.

$h = 8$ cm

$h = 12$ cm

$b = 10$ cm

$b = 15$ cm

30. **MUSIC** You can buy 3 CDs for $52.20 or 5 CDs for $62.45. Are the rates proportional? Explain.

31. **BASEBALL** The table shows pitching statistics for four pitchers during the 2008 season.

    a. Which pitcher has the highest ratio of strikeouts to walks?

    b. Which of the pitchers have equivalent strikeout to walk ratios?

2008 Season		
Pitcher	**Strikeouts**	**Walks**
Pitcher 1	6	8
Pitcher 2	8	4
Pitcher 3	10	1
Pitcher 4	10	5

32. **NAIL POLISH** A specific shade of red nail polish requires 7 parts red to 2 parts yellow. A mixture contains 35 quarts of red and 8 quarts of yellow. How can you fix the mixture to make the correct shade of red?

33. **COIN COLLECTION** The ratio of quarters to dimes in a coin collection is $5:3$. The same number of new quarters and dimes are added to the collection.

    a. Is the ratio of quarters to dimes still $5:3$?

    b. If so, illustrate your answer with an example. If not, show why with a "counterexample."

34. **Critical Thinking** Ratio A is equivalent to ratio B. Ratio B is equivalent to ratio C. Is ratio A equivalent to ratio C? Explain.

Fair Game Review *What you learned in previous grades & lessons*

Add or subtract. *(Sections 1.2 and 1.3)*

35. $-28 + 15$

36. $-6 + (-11)$

37. $-10 - 8$

38. $-17 - (-14)$

39. **MULTIPLE CHOICE** Which fraction is not equivalent to $\dfrac{2}{6}$? *(Skills Review Handbook)*

    (A) $\dfrac{1}{3}$

    (B) $\dfrac{12}{36}$

    (C) $\dfrac{4}{12}$

    (D) $\dfrac{6}{9}$